8.5 Calculations Involving Basic Solutions

In Section 8.4 we explored a series of calculations for strong and weak acids. These calculations have parallels for strong and weak bases. In general, the calculations are done in exactly the same way, but with the relevant base-related values (K_b, $[OH^-(aq)]$, pOH) substituted for the acid-related values (K_a, $[H^+(aq)]$, pH).

Calculations Involving Solutions of Strong Bases

The pOH of strong bases depends entirely on the $[OH^-(aq)]$ contributed by the ionization of the base solute in water. We may ignore the small contribution made by the autoionization of water. Also, we may assume that the metal cation produced by the ionization of a strong base has no effect on the pH of the solution.

As we have already seen, we may calculate the pOH of a basic solution from the solution's pH by applying the equation

$$\text{pH} + \text{pOH} = 14$$

We may also determine solution characteristics for strong bases from the solution concentration, as we did for acids. Tutorial 1 provides examples.

Tutorial 1 Determining $[H^+(aq)]$, $[OH^-(aq)]$, and pH

In this tutorial, you will calculate the $[H^+(aq)]$ and $[OH^-(aq)]$ in a solution of a strong base, given the concentration of the base.

Sample Problem 1: Calculating $[H^+(aq)]$ and $[OH^-(aq)]$ from Solution Concentration

A chemist mixes a 0.025 mol/L solution of barium hydroxide, $Ba(OH)_2(aq)$, a strong base. Calculate $[H^+(aq)]$ and $[OH^-(aq)]$ in this solution.

Given: $[Ba(OH)_2(aq)] = 0.025$ mol/L

Required: $[H^+(aq)]$ and $[OH^-(aq)]$

Solution:

Since $Ba(OH)_2(aq)$ is a strong base, it dissociates completely into its ions. First write the dissociation equation:

$$Ba(OH)_2(aq) \rightarrow Ba^{2+}(aq) + 2\ OH^-(aq)$$

Since every mole of $Ba(OH)_2(aq)$ produces 1 mol of $Ba^{2+}(aq)$ and 2 mol of $OH^-(aq)$,

$$[OH^-(aq)] = 2(0.025\ \text{mol/L})$$
$$[OH^-(aq)] = 0.050\ \text{mol/L}$$

While the autoionization of water produces hydroxide ions, this contribution is insignificant when compared to the 0.050 mol/L of ions produced by the dissociation of $Ba(OH)_2(aq)$. Also, since $Ba(OH)_2$ is a strong base, you may assume that the $Ba^{2+}(aq)$ ions do not affect the acid–base properties of the solution. In general, you may ignore the presence of the cations of all ionic hydroxides when determining acid–base properties of solutions.

Since $Ba(OH)_2(aq)$ is a strong base, it dissociates completely. The only equilibrium system that you need to consider in this problem is the autoionization of water. This reaction is important because it produces $H^+(aq)$, which affects pH.

$$H_2O(l) \rightleftharpoons H^+(aq) + OH^-(aq) \quad K_w = 1.0 \times 10^{-14}$$

The additional OH^- (aq) ions produced shift this equilibrium to the left, reducing the $[H^+(aq)]$ and thus raising the pH. Use the K_w equation to determine $[H^+(aq)]$:

$$K_w = [H^+(aq)][OH^-(aq)]$$
$$[H^+(aq)] = \frac{K_w}{[OH^-(aq)]}$$
$$= \frac{1.0 \times 10^{-14}}{0.050}$$
$$[H^+(aq)] = 2.0 \times 10^{-13}\ \text{mol/L}$$

Notice that the $[OH^-(aq)]$ used was 0.050 mol/L. Although some $OH^-(aq)$ ions react with $H^+(aq)$ ions to form $H_2O(l)$ when the equilibrium shifts to the left, the amount of $OH^-(aq)$ ions that react is so small that the $[OH^-(aq)]$ essentially remains at 0.050 mol/L.

Statement: The concentration of hydrogen ions in the barium hydroxide solution is 2.0×10^{-13} mol/L. The concentration of hydroxide ions is 0.050 mol/L.

Sample Problem 2: Calculating pH from Solution Concentration

Determine the pH of a 500.0 mL solution produced by dissolving 2.6 g of sodium hydroxide, NaOH(s), in water.

Given: $m_{\text{NaOH}} = 2.6$ g; $V = 500.0$ mL

Required: pH

Solution:

Step 1. Write the balanced equation for the dissociation of the base.

$$\text{NaOH(aq)} \rightarrow \text{Na}^+\text{(aq)} + \text{OH}^-\text{(aq)}$$

Step 2. Identify the major entities in solution at equilibrium.

The major entities in solution are Na^+(aq), OH^-(aq), and H_2O(l).

Step 3. Determine $[\text{OH}^-\text{(aq)}]$.

Use the mass of NaOH(s) and the volume of the solution to calculate the concentration of the base before dissociation.

$$m_{\text{NaOH}} = 2.6 \text{ g}$$

$$M_{\text{NaOH}} = 40.00 \text{ g/mol}$$

$$n_{\text{NaOH}} = 2.6 \cancel{\text{g}} \times \frac{1 \text{ mol}}{40.00 \cancel{\text{g}}}$$

$$n_{\text{NaOH}} = 0.065 \text{ mol}$$

$$c_{\text{NaOH}} = \frac{0.065 \text{ mol}}{0.5000 \text{ L}}$$

$$c_{\text{NaOH}} = 0.13 \text{ mol/L}$$

From the balanced equation, note that every mole of sodium hydroxide dissociates into 1 mol of Na^+(aq) and 1 mol of OH^-(aq). Therefore, after dissociation,

$$[\text{OH}^-\text{(aq)}] = 0.13 \text{ mol/L}$$

Step 4. Determine $[\text{H}^+\text{(aq)}]$.

Use the K_w equation and your value of $[\text{OH}^-\text{(aq)}]$ to find $[\text{H}^+\text{(aq)}]$.

$$K_w = [\text{H}^+\text{(aq)}][\text{OH}^-\text{(aq)}]$$

$$[\text{H}^+\text{(aq)}] = \frac{K_w}{[\text{OH}^-\text{(aq)}]}$$

$$= \frac{1.0 \times 10^{-14}}{0.13} \text{ mol/L}$$

$$[\text{H}^+\text{(aq)}] = 7.7 \times 10^{-14} \text{ mol/L}$$

Step 5. Determine pH.

$$\text{pH} = -\log[\text{H}^+\text{(aq)}]$$

$$= -\log(7.7 \times 10^{-14})$$

$$\text{pH} = 13.11$$

Statement: The pH of the solution is 13.11.

Practice

1. Calculate the $[\text{H}^+\text{(aq)}]$ and $[\text{OH}^-\text{(aq)}]$ in a 0.00100 mol/L solution of potassium hydroxide, KOH(aq). K/U [ans: $[\text{OH}^-\text{(aq)}] = 0.00100$ mol/L; $[\text{H}^+\text{(aq)}] = 1.00 \times 10^{-11}$ mol/L]
2. Calculate the pH of a 2.00 L solution prepared by dissolving 0.042 mol of strontium hydroxide, Sr(OH)_2, in water. K/U [ans: 3.92]

Calculations Involving Solutions of Weak Bases

In Section 8.4 you used the acid ionization constant, K_a, with pH and $[\text{H}^+\text{(aq)}]$ in calculations involving weak acids. In much the same way, you may use the base ionization constant, K_b, with pOH and $[\text{OH}^-\text{(aq)}]$ in calculations involving weak bases.

When solving problems involving weak bases, you will need to know the value of K_b for the base in question. In Section 8.2 you learned how to find the K_b value of a base when given the K_a value of its conjugate acid. Sometimes, however, you may not be given either value. In this situation you may find the K_a value in a table (Table 1 in Appendix B5), and then calculate K_b.

Tutorial 2 Calculating K_b and pH for a Weak Base

This tutorial outlines how to find the K_b value for a base when the K_a value of the conjugate acid is not given, and to use this value to determine pH.

Sample Problem 1: Determining K_b from K_a and pH from Concentration

Highly concentrated ammonia solutions used by industry are sometimes called "liquor ammonia." These solutions must be handled carefully as they are corrosive and toxic (**Figure 1**). Calculate the pH of a 15.0 mol/L solution of ammonia, NH_3(aq).

Figure 1 This industrial facility in Saskatchewan produces both ammonia and urea for use as fertilizers and in other applications.

Given: $[NH_3(aq)] = 15.0$ mol/L

Required: pH

Solution:

Step 1. Determine the formula of the base's conjugate acid.

The conjugate acid of ammonia is the ammonium ion, $NH_4^+(aq)$.

Step 2. Look up the conjugate acid's K_a value (Table 1, Appendix B5).

K_a for $NH_4^+(aq)$ is 5.8×10^{-10}.

Step 3. Rearrange the equation $K_aK_b = K_w$ to solve for K_b. Then, substitute in the values for K_a and K_w, noting that $K_w = 1.0 \times 10^{-14}$, and solve the equation.

$$\begin{aligned} K_b &= \frac{K_w}{K_a} \\ &= \frac{1.0 \times 10^{-14}}{5.8 \times 10^{-10}} \\ K_b &= 1.724 \times 10^{-5} \text{ (2 extra digits carried)} \end{aligned}$$

Step 4. Identify the major entities in solution at equilibrium.

Since ammonia is a weak base, most of the dissolved ammonia will remain as $NH_3(aq)$. Thus the major entities in solution are $NH_3(aq)$ and $H_2O(l)$.

Step 5. Identify the primary source of hydroxide ions in solution.

$NH_3(aq)$ and $H_2O(l)$ can produce $OH^-(aq)$, as shown below.

$$NH_3(aq) + H_2O(l) \rightleftharpoons NH_4^+(aq) + OH^-(aq) \qquad K_b = 1.7 \times 10^{-5}$$

$$H_2O(l) \rightleftharpoons H^+(aq) + OH^-(aq) \qquad K_w = 1.0 \times 10^{-14}$$

Since $K_b >> K_w$, the contribution of water to $[OH^-(aq)]$ can be ignored.

Step 6. Write the equilibrium constant equation for the reaction that is the primary source of $OH^-(aq)$ ions:

$$K_b = \frac{[NH_4^+(aq)][OH^-(aq)]}{[NH_3(aq)]}$$

$$1.724 \times 10^{-5} = \frac{[NH_4^+(aq)][OH^-(aq)]}{[NH_3(aq)]}$$

Step 7. Determine initial and equilibrium concentrations.

Set up an ICE table for the ionization of $NH_3(aq)$ (**Table 1**).

Table 1 ICE Table for the Ionization of $NH_3(aq)$

	NH_3(aq)	$\rightleftharpoons$	OH^-(aq)	+	NH_4^+(aq)
I	15.0		0		0
C	$-x$		$+x$		$+x$
E	$15.0 - x$		x		x

Step 8. Substitute values from the ICE table into the equilibrium constant equation:

$$K_b = \frac{[NH_4^+(aq)][OH^-(aq)]}{[NH_3(aq)]}$$

$$K_b = \frac{(x)(x)}{15.0 - x}$$

Step 9. Use the hundred rule to determine whether any simplifying assumptions may be made.

$$\frac{[NH_3(aq)_{initial}]}{K_b} = \frac{15.0}{1.724 \times 10^{-5}}$$

$$\frac{[NH_3(aq)_{initial}]}{K_b} = 8.7 \times 10^{5}$$

Since the $\frac{[NH_3(aq)_{initial}]}{K_b}$ ratio is much greater than 100, you may assume that

$15.0 - x \approx 15.0$

Thus, the equilibrium equation is

$$1.724 \times 10^{-5} \approx \frac{x^2}{15.0}$$

Step 10. Solve for *x*.

$$1.724 \times 10^{-5} \approx \frac{x^2}{15.0}$$

$$x^2 \approx (1.724 \times 10^{-5})(15.0)$$

$$x^2 \approx 2.6 \times 10^{-4}$$

$$x \approx \sqrt{2.6 \times 10^{-4}}$$

$$x \approx 1.612 \times 10^{-2}$$

Step 11. Use the 5 % rule to check your result.

$$\frac{x}{[NH_3(aq)]} \times 100\ \% = \frac{1.724 \times 10^{-5}}{15.0} \times 100\ \%$$

$$\frac{x}{[NH_3(aq)]} \times 100\ \% = 0.00011\ \%$$

Since 0.00011 % < 5 %, the error in this result is acceptable.

Step 12. Determine the $[OH^-(aq)]$ at equilibrium, then calculate pOH and pH.

Since $[OH^-(aq)] = x$ (from the ICE table) and $x \approx 1.612 \times 10^{-2}$

$$[OH^-(aq)] \approx 1.612 \times 10^{-2}\ \text{mol/L}$$

$$\text{pOH} = -\log[OH^-(aq)]$$

$$\approx -\log(1.612 \times 10^{-2})$$

$$\text{pOH} \approx 1.79$$

$$\text{pH} + \text{pOH} = 14$$

$$\text{pH} = 14 - \text{pOH}$$

$$\approx 14 - 1.79$$

$$\text{pH} \approx 12.21$$

Statement: The pH of the ammonia solution is 12.21.

Practice

1. Determine K_b for
   (a) the ethanoate ion, $C_2H_3O_2^-(aq)$ [ans: 5.6×10^{-10}]
   (b) the borate ion, $H_2BO_3^-(aq)$ K/U [ans: 1.7×10^{-5}]
2. Calculate the pH of a 0.20 mol/L solution of a base, where $K_b = 3.82 \times 10^{-10}$. K/U [ans: 8.94]
3. Determine the pH of a 4.5 mol/L solution of hydrazine, $N_2H_4(aq)$. Look up the K_b value for hydrazine. K/U [ans: 11.44]

UNIT TASK **BOOKMARK**

If the consumer product that you have chosen for the Unit Task on page 582 is a base, how does the high pH affect its function?

8.5 Review

Summary

- Calculations to determine solution characteristics of weak bases are generally similar to those for determining the characteristics of weak acid solutions.
- The value for K_a can often be found in a reference table. The value for K_b can then be calculated from K_a.

Questions

1. Calculate the pH of each of the following solutions. If necessary, look up the K_b values in Table 3, Appendix B5. K/U
    (a) sodium hydroxide, $NaOH(aq)$; $[NaOH(aq)] = 0.0030$ mol/L
    (b) barium hydroxide, $Ba(OH)_2(aq)$; $[Ba(OH)_2(aq)] = 0.0020$ mol/L
    (c) dimethylamine, $(CH_3)_2NH(aq)$; $[(CH_3)_2NH(aq)] = 0.010$ mol/L
    (d) hydrazine, $N_2H_4(aq)$; $[N_2H_4(aq)] = 0.0250$ mol/L
2. Determine the K_b value for each of the following substances. (For some of them you will have to first look up the K_a values in Table 1 in Appendix B5.) K/U
    (a) hydrogen sulfide ion, $HS^-(aq)$
    (b) nitrite ion, $NO_2^-(aq)$; $K_a = 7.2 \times 10^{-4}$
    (c) cyanide ion, $CN^-(aq)$
    (d) fluoride ion, $F^-(aq)$
3. Calculate the pH of the following solutions at SATP: K/U
    (a) morphine, $C_{17}H_{19}NO_3(aq)$; $[C_{17}H_{19}NO_3(aq)] = 0.01$ mol/L; $K_b = 7.5 \times 10^{-7}$
    (b) strychnine, $C_{21}H_{22}N_2O_2(aq)$; $[C_{21}H_{22}N_2O_2(aq)] = 0.001$ mol/L; $K_b = 1.0 \times 10^{-6}$
4. For a 0.20 mol/L solution of each of the following bases, calculate the hydroxide ion concentration, hydrogen ion concentration, and pH: K/U
    (a) *N,N*-diethylethanamine, $(C_2H_5)_3N(aq)$; $K_b = 4.0 \times 10^{-4}$
    (b) hydroxylamine, $HONH_2(aq)$; $K_b = 1.1 \times 10^{-8}$
5. Calculate $[OH^-(aq)]$, pOH, and pH for a 0.000 40 mol/L solution of calcium hydroxide, $Ca(OH)_2(aq)$; $K_b = 5.0 \times 10^{-11}$ K/U
6. Calculate $[OH^-(aq)]$, pOH, and pH for each of the following aqueous solutions: K/U
    (a) 25 g potassium hydroxide, $KOH(aq)$, per litre
    (b) 150.0 g sodium hydroxide, $NaOH(aq)$, per litre
7. Ammonia is the most common and well-known weak base. A sample of household ammonia has a pH of 11.80. Determine its concentration. K/U A
8. Find an online MSDS for ammonia. Prepare a list of precautions that should be taken while working with ammonia. C A
9. Methanoate ion, $HCO_2^-(aq)$, is the conjugate base of methanoic (formic) acid, $HCO_2H(aq)$. K/U
    (a) Write the K_b equation for the ionization of the methanoate ion.
    (b) Write the K_a equation for the ionization of methanoic acid.
    (c) Use the equations from (a) and (b) to show that $K_aK_b = K_w$.
10. Methylamine, $CH_3NH_2(aq)$, is a weak base ($K_b = 4.4 \times 10^{-4}$). It is used in the pharmaceutical industry to synthesize ephedrine, which is a stimulant, decongestant, and appetite suppressant. T/I C A
    (a) A pharmaceutical company requires a solution of 0.25 mol/L methylamine for its reactions. One of the chemists onsite was unsure if the methylamine supplied was of the correct concentration. He tested the pH and determined it was 11.2. Did this sample of methylamine have the concentration required for their reaction? Show your evidence.
    (b) If you determined that the solution was not the correct concentration, was it too concentrated or too dilute?
    (c) Ephedrine is one of the main ingredients in a "Coast Guard cocktail." Research why ephedrine is used in this application, and the possible health effects of its use.

WEB LINK